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ANALYSIS OF THE TRANSMISSION PROPERTIES OF GROUNDED FINLINES ON ANISOTROPIC SUBSTRATES

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Abstract

This contribution describes a rigorous method for an efficient computation of finlines on anisotropic substrates. It allows a realistic description of these waveguides because it can also consider the second order effects such as the influence of the metallization thickness and of the substrate grooves on the transmission properties, which are calculated by means of integral eigenvalue equations generated for the interfaces of the considered cross-section. Several examples for the effective dielectric constant illustrate the applicability of the described method.

1 Introduction

Up to now, several contributions have been published describing the transmission properties of microstrip transmission lines on anisotropic substrates <1,6,7>. In opposition to the microstrip lines, finlines on anisotropic substrates have hardly been treated.
In the following, a calculation method for all known grounded finline configurations is described which takes into account both the finite metallization thickness and the influence of the longitudinal slits in the waveguide mount <2>. Without restrictions of universal validity, the method presented in this contribution will be demonstrated to the unilateral finline.

2 The Eigenvalue Problem of the Unilateral Finline on Anisotropic Substrates

Let Fig. 1 represent the cross-section of an unilateral finline on anisotropic substrate with ideal conducting metallic walls and metallizations. In order to treat of the above eigenvalue problem a complete hybrid mode analysis is necessary. All the electromagnetic fields inside the considered waveguide have the same z-coordinate dependence of $\exp(\pm\gamma z)$ with γ the propagation constant. The harmonic time dependence together with this z-dependence will be omitted in the following train of thoughts. It will also be assumed that there are no free sources in the inner region being considered.

Now, the cross-section of the unilateral finline which is given by Fig. 1, will be subdivided into four regions (1) - (4).

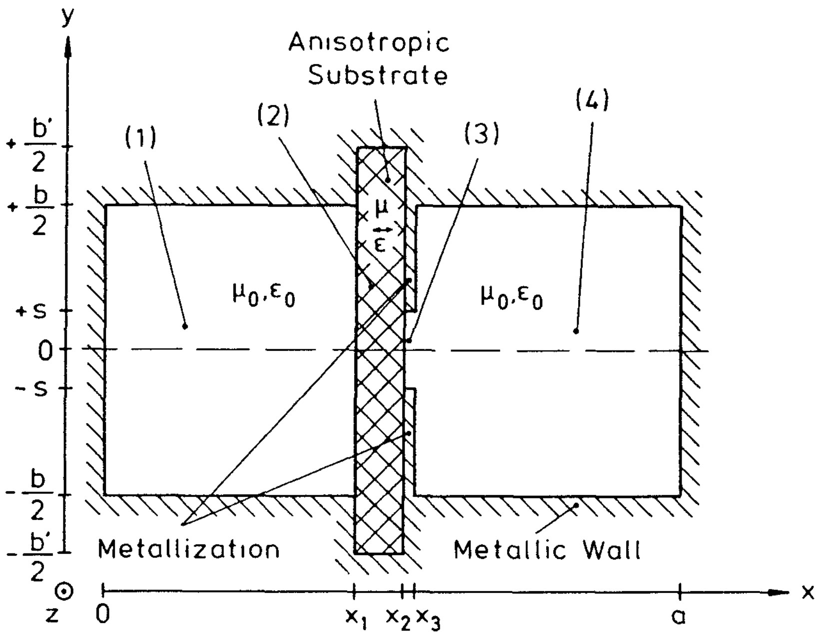

Fig. 1: Cross-section of an unilateral finline which to be analyzed.

The Hertzian potentials Φ_x and Ψ_x satisfy the two dimensional Helmholtz equations in regions (1), (3) and (4) as well as the boundary conditions on the metallic walls, while these only inside the slot and not on the metallization can be fulfilled.

With reference to Fig. 1, for the region (1) it is appropiate to write:

$$\Phi_x^{(1)} = \sum_{m=1}^{\infty} B_m^{(1)} \cos(k_{xm}^{(1)} x) \sin(k_{ym}^{(1)} y) , \quad (1)$$

$$\Psi_x^{(1)} = \sum_{m=1}^{\infty} A_m^{(1)} \sin(k_{xm}^{(1)} x) \cos(k_{ym}^{(1)} y) , \quad (2)$$

where

$$k_{ym}^{(1)} = \frac{m\pi}{b} \quad . (3)$$

The TE-to-x and TM-to-x field components for the region (1) are derivable from the Hertzian potentials $\Phi_x^{(1)}$ and $\Psi_x^{(1)}$ respectively. It can easily be shown that for the subregions (3) and (4) analogous Hertzian potentials are valid. The separation parameter equations in the regions (1), (3) and (4) are

$$k_{xm}^{(i)\,2} + k_{ym}^{(i)\,2} - \gamma^2 = k_0^2 \quad , i=1,3,4 \quad (5)$$

where

$$k_0^2 = \varepsilon_0 \mu_0 \omega^2 \quad . (6)$$

If the problem of the finlines on anisotropic substrates is to be solved, it is a fundamental supposition that the electromagnetic fields of this material (s. region (2)) are known, too. The next section descibes the field theory that may be used to determine the electromagnetic fields of the anisotropic substrate.

3 Electrocmagnetic Fields in Anisotropic Substrates

A complete study of anisotropic slabs in rectangular waveguides is given by <4>, where the matrix representation of fields is used. This method can be developed and matched to the requirements of finlines considered here.

It is assumed that the electromagnetic properties of the lossless anisotropic substrate (subregion (2)) could be described by the permittivity tensor of second stage <1>:

$$\overleftrightarrow{\varepsilon} = \varepsilon_0 \begin{bmatrix} \varepsilon_{xx} & 0 & \varepsilon_{xz} \\ 0 & \varepsilon_{yy} & 0 \\ \varepsilon_{zx} & 0 & \varepsilon_{zz} \end{bmatrix} \quad . (7)$$

Therefore, in subregion (2) the fields are expanded into the following terms <4>:

$$E_n^{(2)} = \sum_{m=0}^{\infty} C_{nm}^{(2)}(x) \begin{Bmatrix} \sin \\ \cos \end{Bmatrix} (k_{ym}^{(2)} (y-b'/2)) \quad (8)$$

and

$$H_n^{(2)} = \sum_{m=0}^{\infty} K_{nm}^{(2)}(x) \begin{Bmatrix} \sin \\ \cos \end{Bmatrix} (k_{ym}^{(2)} (y-b'/2)) \quad (9)$$

with n = x, y, z and

$$k_{ym}^{(2)} = \frac{m\pi}{b'} \quad . (10)$$

From the coupled differential equations

$$\left\langle \frac{d^2}{dx^2} + f_1(\gamma) \right\rangle K_{ym}^{(2)} + c_1 C_{ym}^{(2)} = 0 \quad , (11)$$

$$\left\langle \frac{d^2}{dx^2} + f_2(\gamma) \right\rangle C_{ym}^{(2)} + c_2 K_{ym}^{(2)} = 0 \quad , (12)$$

the unknown function $C_{ym}^{(2)}$ and $K_{ym}^{(2)}$ can be determined as linear combinations of sine- and cosine-functions. The function f_1 and f_2 as well as the constants c_1 and c_2 are composed by the tensor elements and propagation constants and can be derived from the Maxwell's equations. As an example the function $C_{ym}^{(2)}$ is given by

$$C_{ym}^{(2)}(x) = C_{1ym}^{(2)} \cos(k_{xm}^{(2)} (x-x_1)) + C_{2ym}^{(2)} \sin(k_{xm}^{(2)} (x-x_1)) \quad . (13)$$

By utilization of the solutions for $C_{ym}^{(2)}$ and $K_{ym}^{(2)}$, the other functions

$C_{xm}^{(2)},\ldots,\ K_{xm}^{(2)},\ldots$ can also be determined. Using the results for the functions $C_{xm}^{(2)},\ldots,\ K_{xm}^{(2)},\ldots,$ the field components of the electromagnetic field can be derived (s. eqs. (8) and (9)).

4 The Eigenvalue Equation

The total fields obtained from the Hertzian potentials and from the theory described above must satisfy the interface conditions at $x = x_1$, $x = x_2$ and $x = x_3$ (Fig. 1). Considering the interface $x = x_2$, there are the following eight boundary and continuity conditions which are mutally independent:

$$\begin{array}{lll} -b'/2<y<-s: & E_y^{(2)} = 0 & , \\ & E_z^{(2)} = 0 & , \\ -s<y<+s: & E_y^{(2)} = E_y^{(3)} & , \\ & E_z^{(2)} = E_z^{(3)} & , \\ & H_y^{(2)} = H_y^{(3)} & , \\ & H_z^{(2)} = H_z^{(3)} & , \\ +s<y<+b'/2: & E_y^{(2)} = 0 & , \\ & E_z^{(2)} = 0 & . \end{array} \qquad (14)$$

It can be shown that at the interfaces $x = x_1$, resp. $x = x_3$ similar boundary and continuity conditions are valid. It is in satisfying all these conditions mentioned above that different mathematical procedures are employed leading to different forms of the eigenvalue equation determining the transmission properties of the finline. One of these mathematical procedures is the Ritz-Galerkin method which allows the elimination of the y-dependence of the boundary and continuity conditions. Using this method as described for finlines with isotropic substrates in <2> yields a system of integral eigenvalue equations which can be converted into a system of homogeneous equations:

$$\overleftrightarrow{F} \cdot \vec{A} = \vec{0} \qquad (15)$$

with $\overleftrightarrow{F}$ the system matrix and $\vec{A}$ the amplitude vector. This eigenvalue equation can only be solved in a nontrivial way if eq. (16) is valid:

$$\det\{\overleftrightarrow{F}(\gamma)\} \overset{!}{=} 0 \qquad (16)$$

thus yielding the propagation constant γ and the field distributions.

5 Results and Discussion

Consider the unilateral finline in Fig. 1 on an anisotropic substrate whose permittivity tensor is given by eq. (7). In dimensioning such a waveguide, the effective dielectric constant must be known. This value can be obtained, e.g., from the ridge-loaded waveguide model for grounded finlines <2>. In this case, the effective dielectric constant of the unilateral finline is calculated from the operating frequency, wavelength and cutoff wavelength of an equivalent air-filled ridge-loaded rectangular waveguide.

Another possibility to define the effective dielectric constant is, if the phase constant is quadratic normalized on the wave number k_0:

$$\varepsilon_{eff} = (\beta \,/\, k_0)^2 \qquad . (17)$$

Using the results for the fields from the method described above, the effective dielectric constant of the cross-section (s. Fig. 1) should be calculated by eq. (17).

Tab. 1 shows the comparison of the effective dielectric constant for isotropic and similar anisotropic substrate materials in dependence of the slot width $2\cdot s$.

ε_{eff}	isotropic	anisotropic
$2\cdot s$ = 0.6 mm	1.17	1.32
$2\cdot s$ = 0.8 mm	1.11	1.23
$2\cdot s$ = 1.0 mm	1.02	1.12

Tab. 1: Effective dielectric constant for an unilateral finline on isotropic substrate with $\varepsilon_r = 2.77$ and an unilateral finline on anisotropic substrate with anisotropic PTFE cloth rotated by 30°. The tensor elements results as: $\varepsilon_{xx} = 2.77$, $\varepsilon_{yy} = 2.88$, $\varepsilon_{zz} = 2.54$, $\varepsilon_{xz} = \varepsilon_{zx} = -0.19$. (a=7.112mm, b=3.556mm, b'=4.056mm, thickness of the substrate = 250µm, thickness of the metallization = 50µm).

Tab. 2 shows that especially for finlines with large slots it is important to take the finite slit depth into account. Errors of 3.6 percent (Ka-range values) in the effective dielectric constant of the unilateral finline on anisotropic substrate can be realized by neglecting the effects of the longitudinal slots. For the isotropic finline the error is about 4.9 percent. This error is greater than for the anisotropic finline because the field concentration in the slot region is proportional to the effective dielectric constant. Therefore the influence of substrate slits in the finline mount decrease for higher effective permittivities.

ε_{eff}	isotropic	anisotropic
b' = 3.556 mm	1.02	1.12
b' = 4.056 mm	0.97	1.08

Tab. 2: Effective dielectric constant for an unilateral finline on isotropic substrate with $\varepsilon_r = 2.77$ and an unilateral finline on anisotropic substrate with anisotropic PTFE cloth rotated by 30°. The tensor elements results as: $\varepsilon_{xx} = 2.77$, $\varepsilon_{yy} = 2.88$, $\varepsilon_{zz} = 2.54$, $\varepsilon_{xz} = \varepsilon_{zx} = -0.19$. (a=7.112mm, b=3.556mm, 2.s=1mm, thickness of the substrate = 250µm, thickness of the metallization = 50µm).

The trends of these results obtained above are identical with those by <2> derived for finlines on isotropic substrates.

6 Conclusion

As an example, the unilateral finline on anisotropic substrate has been analyzed using the orthogonal series field representation technique and the Ritz-Galerkin method. It has also been demonstrated the effects of substrate anisotropy on the unilateral finline characteristics. The method employed here renders possible the consideration of the second order effects, too. This technique is numerically very advantageous, since it only needs a small number of geometrical and electrical parameters and amplitude coefficients.

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